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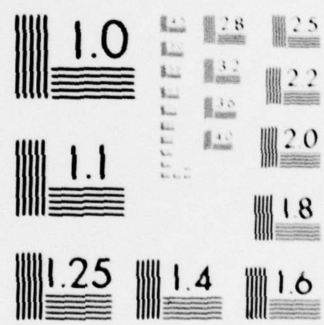

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INTERVAL ESTIMATION AFTER SEQUENTIAL TESTING
FOR THE MEAN OF THE EXPONENTIAL DISTRIBUTION
IN THE LARGE SAMPLE CASE*

by

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ABSTRACT

Let m items be put on test at the outset, and suppose an item is not replaced upon failure. Assume an exponential failure distribution $F_\theta(t) = 1 - \exp(-t/\theta)$. A time truncated sequential procedure for testing H_0: $\theta \geq \theta_0$ versus H_1: $\theta \leq \theta_1$ is developed. This procedure allows a quick rejection of H_0 when H_1 is true, but provides an accurate interval estimate of θ when H_0 is accepted after the test has been established.

AMS Subject Classification: Primary 62L10

Key Words and Phrases: Time truncation; Gaussian process; Wiener measure; Weak convergence; Stopping rule; Confidence interval; Average sampling time.

1. INTRODUCTION

In the quality control of industrial production, it is a common practice to test hypotheses to make a decision about acceptance or rejection of a production batch. Generally, a sequential testing procedure requires less data to reach a decision than a fixed sample or fixed time testing procedure. Also, in many situations, the experimenter may want to obtain a point or an interval estimate of some parameter after a test of hypothesis has been established. (for example, see Siegmund (1977,1978)).

In this paper, we will consider a commonly used life testing procedure in industrial settings. Let m items be independently put on test at the outset, and when an item fails, it is not replaced. Suppose that the failure time is exponentially distributed with distribution function $F_\theta(t) = 1-\exp(-t/\theta)$, $t \geq 0$. The hypotheses which are to be tested are

$$H_0: \theta \geq \theta_0 \text{ versus } H_1: \theta \leq \theta_1, \text{ where } \theta_1 < \theta_0. \tag{1.1}$$

We assume that for certain reasons this life test must be terminated at time t_0 if not all of the items have failed by t_0. (This type of censorship is often called Type I censoring.) The time t_0 can be preassigned or determined by the specified probabilities α and β for testing (1.1).

The maximum likelihood estimator of θ under the above type of censorship with censoring time $t(\leq t_0)$ was given by Bartlett (1953) as

$$\hat{\theta}_m(t) = \frac{\sum_{i=1}^{N_m(t)} U_i + (m-N_m(t))t}{N_m(t)},$$

where $N_m(t)$ denotes the number of failures at or before time t and the

U_i's denote the observed failure times in increasing order. For convenience, let us define $\hat{\theta}_m(t) = \infty$ if $N_m(t) = 0$. Bartholomew (1963) obtained the conditional probability that $\hat{\theta}_m(t) \geq c$ given $N_m(t) > 0$, when $c > 0$. Yang and Sirvanci (1977) showed that the estimator $\hat{\theta}_m(t)$ is consistent and, when properly standardized, is asymptotically normal for each fixed $t \leq t_0$. Recently, Spurrier and Wei (1979) have used $\hat{\theta}_m(t_0)$ as the test statistic for testing (1.1) and demonstrated certain advantages over Epstein's procedure (1954) and the pure sequential probability ratio test given in Ghosh (1970, pp. 193-96). Note that Spurrier and Wei's test is a test based on the fixed length of time t_0.

In this article, a one-sided sequential test procedure based on the random function $\hat{\theta}_m$ is presented for testing (1.1). This procedure allows a quick rejection of H_0 when H_1 is true, but provides an accurate interval estimator of θ when H_0 is accepted. Siegmund (1977) studied the same problem for different experimental settings. In Section 2, we prove that the standardized process $\hat{\theta}_m$ converges in distribution to a Gaussian process as $m \to \infty$. The large sample approximation of the power function and the average sampling time (AST) are presented in Sections 3 and 4, respectively. An interval estimate of θ after testing is given in Section 5. A simple example is provided for illustration purposes in Section 6.

2. LARGE SAMPLE APPROXIMATION OF $\hat{\theta}_m$ BY A GAUSSIAN PROCESS

To test the hypothesis (1.1), we define the following stopping rule: Let T be the smallest value of t such that $\hat{\theta}_m(t) \leq c_m$. Given t_0, stop testing at $\min(t_0,T)$ and reject H_0 if and only if $T \leq t_0$.

We now prove that the distribution of the standardized $\hat{\theta}_m$ converges weakly to a Gaussian measure.

Theorem 1. Consider $\hat{\theta}_m(t)$ to be a function of $t \in (0,t_0]$. Then $m^{\frac{1}{2}}(\hat{\theta}_m - \theta)$ converges in distribution to a Gaussian process Y as $m \to \infty$, where $EY(t) = 0$ and $\mathrm{cov}(Y(s),Y(t)) = \theta^2/F_\theta(\max(s,t))$, $s,t \in (0,t_0]$.

Proof. The total time on test at time t is given by

$$\psi_m(t) = \sum_{i=1}^{N_m(t)} U_i + (m-N_m(t))t.$$

First, we prove that the random element $\xi_m = m^{-\frac{1}{2}}(\psi_m - \theta N_m)$ in $D[0,t_0]$ converges in distribution to a Gaussian process Z with $EZ(t) = 0$ and $\mathrm{cov}(Z(s),Z(t)) = \theta^2 F_\theta(\min(s,t))$ as $m \to \infty$, where $s,t \in [0,t_0]$. (See Billingsley (1968) for notation.)

Let $0 \le s_1 \le s_2 \le \ldots \le s_k \le t_0$, where k is an arbitrary positive integer. Also, let $X_1, X_2, \ldots, X_m$ be the hypothetical data from the exponential distribution with mean θ. Note that because of the time truncation t_0 not every X can be observed. Define

$$W_i^{(j)} = (X_i - (s_j+\theta))I_{[s_{j-1} \le X_i < s_j]} + (s_j - s_{j-1}) - (s_j - s_{j-1})I_{[X_i < s_{j-1}]},$$

where $I_{[\cdot]}$ denotes the indicator function, $i = 1,\ldots,n$, and $j = 1,\ldots,k$. It can be shown that $EW_i^{(j)} = 0$, $\sigma_1^2 = \mathrm{var}(W_i^{(1)}) = \theta^2 F_\theta(s_1)$, $\sigma_j^2 = \mathrm{var}(W_i^{(j)}) = \theta^2(F_\theta(s_j) - F_\theta(s_{j-1}))$, $j \ne 1$, and $E(W_i^{(\ell)} W_i^{(\ell')}) = 0$, $\ell \ne \ell'$. By the Central Limit Theorem, the k-dimensional random vector $m^{-\frac{1}{2}} \sum_{i=1}^{m} \underset{\sim}{W}_i$ converges in distribution to a multivariate normal random vector with mean $\underset{\sim}{0}$ and covariance matrix $\ddagger$ as $m \to \infty$, where $\underset{\sim}{W}_i = (W_i^{(1)},\ldots,W_i^{(k)})'$ and $\ddagger = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2)$, ' denoting matrix transpose.

Let $V_m^{(j)} = \sum_{\ell=1}^{j} \sum_{i=1}^{m} W_i^{(\ell)} = \psi_m(s_j) - \theta N_m(s_j)$ and $\underset{\sim}{V}_m = (V_m^{(1)}, \ldots, V_m^{(k)})'$.

Then $m^{-\frac{1}{2}} \underset{\sim}{V}_m$ converges in distribution to a multivariate normal random vector with mean $\underset{\sim}{0}$ and covariance matrix $\Sigma_1 = (a_{ij})_{k\times k}$ as $m\to\infty$, where

$$a_{ij} = \begin{cases} \theta^2 \; F_\theta(s_j) \quad , \quad j \le i \\ \theta^2 \; F_\theta(s_i) \quad , \quad j \ge i, \; i,j = 1,\ldots,k. \end{cases}$$

Let $0 \le r_1 < r < r_2 \le t_0$. Then $\xi_m(r) - \xi_m(r_1) = m^{-\frac{1}{2}} \sum_{i=1}^{m} \tau_i$ and $\xi_m(r_2) - \xi_m(r) = m^{-\frac{1}{2}} \sum_{i=1}^{m} \phi_i$, where $\tau_i = (X_i - (r+\theta))I_{[r_1 \le X_i < r]} + (r-r_1)$ $-(r-r_1)I_{[X_i < r_1]}$ and $\phi_i = (X_i - (r_2 + \theta))I_{[r \le X_i < r_2]} + (r_2-r) - (r_2-r)I_{[X_i < r]}$.
It is straightforward to show that $E\tau_i = E\phi_i = 0$, $E\tau_i\phi_i = 0$,
$\operatorname{var} \tau_i = \theta^2(\exp(-r_1/\theta)-\exp(-r/\theta))$, $\operatorname{var}\phi_i = \theta^2(\exp(-r/\theta)-\exp(-r_2/\theta))$, and
$E\tau_i^2\phi_i^2 = (r-r_1)^2\theta^2(\exp(-r/\theta)-\exp(-r_2/\theta))$. Now,

$$\begin{aligned} E[(\xi_m(r)-\xi_m(r_1))^2(\xi_m(r_2)-\xi_m(r))^2] &= m^{-2}E[(\sum_{i=1}^{m}\tau_i)^2(\sum_{j=1}^{m}\phi_j)^2] \\ &= m^{-1}E[\tau_1^2\phi_1^2] + m^{-1}(m-1)E[\tau_1^2\phi_2^2] \\ &= m^{-1}(r-r_1)^2\theta^2(\exp(-r/\theta)-\exp(-r_2/\theta)) + m^{-1}(m-1)\operatorname{var}\tau_1 \cdot \operatorname{var}\phi_2 \\ &\le m^{-1}(r_2-r_1)^2\theta^2(\exp(-r_1/\theta)-\exp(-r_2/\theta)) + m^{-1}(m-1)(\exp(-r_1/\theta) \\ &\qquad -\exp(-r_2/\theta))^2\,\theta^4 \\ &\le \theta^4[(r_2-r_1)^2/\theta^2 + (\exp(-r_1/\theta)-\exp(-r_2/\theta))^2] \\ &\le \theta^4[(r_2/\theta-r_1/\theta) + (\exp(-r_1/\theta)-\exp(-r_2/\theta))]^2 \\ &= \theta^4[(F_\theta(r_2)+r_2/\theta) - (F_\theta(r_1)+r_1/\theta)]^2. \end{aligned}$$

Thus, by Theorem 15.6 of Billingsley (1968), the random element ξ_m converges in distribution to a Gaussian process Z with $EZ(t) = 0$ and $\operatorname{cov}(Z(s),Z(t)) = \theta^2F_\theta(\min(s,t))$ as $m\to\infty$. Since the random element mN_m^{-1} converges in probability to a constant function $1/F_\theta$ in $D(0,t_0]$, by Theorem 4.4 and

Corollary 1 of Theorem 5.1 of Billingsley (1968), the random element $mN_m^{-1}\xi_m = m^{\frac{1}{2}}(\hat{\theta}_m - \theta)$ converges in distribution to a Gaussian process Y with $EY(t) = 0$ and $\text{cov}(Y(s),Y(t)) = \theta^2/F_\theta(\max(s,t))$ as $m \to \infty$. ///

Theorem 1 will be used to obtain the approximate power function for the test of (1.1) for large m in the next section, to calculate the average sampling time in Section 4, and to obtain interval estimates of θ after testing in Section 5. First, a standard kind of time transformation for the process Z in the proof of Theorem 1 is needed. This is stated in Lemma 1 for future reference.

Lemma 1. Let Z be a Gaussian process with $EZ(t) = 0$ and $\text{cov}(Z(s),Z(t)) = \theta^2 F_\theta(\min(s,t))$, $s, t \in (0,t_0]$. Also, let $\rho(s) = Z(F_\theta^{-1}(s))/\theta$, where F_θ^{-1} is the inverse function of F_θ, $s \in (0,F_\theta(t_0)]$. Then $\rho(s)$ is a standardized Wiener process.

3. ASYMPTOTIC APPROXIMATION OF POWER FUNCTION

Since $N_m(0) = 0$, by our convention in Section 1, $\hat{\theta}_m(0)$ is defined to be ∞. Also, $F_\theta(0) = 0$. Therefore, we define $Z(0) = b$, where b is an arbitrary positive number which can be determined from the specified error probabilities α and β as a result of Theorem 2.

Theorem 2. When m is large,

$$P_\theta(\text{accept } H_0) = P_\theta(T > t_0) \approx \Phi\left(\frac{b - c_m'(\theta)F_\theta(t_0)}{\theta(F_\theta(t_0))^{\frac{1}{2}}}\right) - \exp\left(\frac{2c_m'(\theta)b}{\theta^2}\right)\Phi\left(-\frac{c_m'(\theta)F_\theta(t_0)+b}{\theta(F_\theta(t_0))^{\frac{1}{2}}}\right),$$

where $c_m'(\theta) = m^{\frac{1}{2}}(c_m - \theta)$ and Φ is the distribution function of the standard normal distribution.

Proof. By Theorem 1, we have for the Gaussian process Z of Lemma 1,

$P_\theta(\text{accept } H_0) = P_\theta(m^{\frac{1}{2}}(\hat{\theta}_m(t) - \theta) > c_m'(\theta), \text{ for all } t \in [0,t_0])$

$\xrightarrow[m\to\infty]{} \quad P_\theta\left(\frac{Z(t)}{F_\theta(t)} > c_m'(\theta), \text{ for all } t \in [0,t_0]\right)$ which by Lemma 1 becomes

$$P_\theta\left(\frac{\theta\rho(s)}{s} > c_m'(\theta)\ , \text{ for all } s \in [0,F_\theta(t_0)]\right)$$

$$= P_\theta(\rho(s) > c_m'(\theta)s/\theta, \text{ for all } s \in [0,F_\theta(t_0)])$$

$$= P_\theta\left(\rho(s) < \frac{b-c_m'(\theta)s}{\theta}\ , \text{ for all } s \in [0,F_\theta(t_0)]\right)$$

$$= \Phi\left(\frac{b-c_m'(\theta)F_\theta(t_0)}{\theta(F_\theta(t_0))^{\frac{1}{2}}}\right) - \exp\left(\frac{2c_m'(\theta)b}{\theta^2}\right)\Phi\left(-\frac{c_m'(\theta)F_\theta(t_0)+b}{\theta(F_\theta(t_0))^{\frac{1}{2}}}\right),$$

where the last equality is obtained from an application of the results on page 348 of Shepp (1966). ///

Therefore, in order to determine the constants c_m and b required for testing (1.1) with the specified error probabilities α and β, we must solve the following two nonlinear equations:

$$\Phi\left(\frac{b-c_m'(\theta_0)F_{\theta_0}(t_0)}{\theta_0(F_{\theta_0}(t_0))^{\frac{1}{2}}}\right) - \exp\left(2\ c_m'(\theta_0)b/\theta_0^{\ 2}\right)\cdot\Phi\left(-\frac{b+c_m'(\theta_0)F_{\theta_0}(t_0)}{\theta_0(F_{\theta_0}(t_0))^{\frac{1}{2}}}\right)$$

$$= 1 - \alpha, \tag{3.1}$$

$$\Phi\left(\frac{b-c_m'(\theta_1)F_{\theta_1}(t_0)}{\theta_1(F_{\theta_1}(t_0))^{\frac{1}{2}}}\right) - \exp\left(2\ c_m'(\theta_1)b/\theta_1^{\ 2}\right)\cdot\Phi\left(-\frac{b+c_m'(\theta_1)F_{\theta_1}(t_0)}{\theta_1(F_{\theta_1}(t_0))^{\frac{1}{2}}}\right) = \beta.$$

A solution to (3.1) may be easily found by numerical analysis techniques. An example is given in Section 6.

4. ASYMPTOTIC APPROXIMATION OF DISTRIBUTION OF T

The approximate distribution of the time T for large m may be readily obtained from the results of Sections 2 and 3. The distribution function of T is given in Theorem 3.

Theorem 3. When m is large,

$$G_\theta(t) = P_\theta(T \le t) = \Phi\left(\frac{c_m'(\theta)F_\theta(t) - b/\theta}{(F_\theta(t))^{\frac{1}{2}}}\right) + \exp\left(\frac{2c_m'(\theta)b}{\theta^2}\right)\Phi\left(\frac{-c_m'(\theta)F_\theta(t) - b/\theta}{(F_\theta(t))^{\frac{1}{2}}}\right),$$

where $t > 0$.

Proof. From Theorem 1, Lemma 2, and equation (17.1) of Shepp (1966),

$$\begin{aligned}
P_\theta(T \le t) &= 1 - P_\theta(T > t) = 1 - P_\theta(\sqrt{m}(\hat\theta_m(u) - \theta) > c_m'(\theta) \text{ for all } u \in [0,t]) \\
&= 1 - P_\theta\left(\frac{Z(u)}{F_\theta(u)} > c_m'(\theta), \text{ for all } u \in [0,t]\right) \\
&= 1 - P_\theta\left(\frac{\theta\rho(v)}{v} > c_m'(\theta), \text{ for all } v \in [0, F_\theta(t)]\right) \\
&= 1 - P_\theta\left(\rho(v) < \frac{b - c_m'(\theta)v}{\theta}, \text{ for all } v \in [0, F_\theta(t)]\right) \\
&= 1 - \left\{\Phi\left(\frac{b - c_m'(\theta)F_\theta(t)}{\theta(F_\theta(t))^{\frac{1}{2}}}\right) - \exp\left(\frac{2c_m'(\theta)b}{\theta^2}\right)\Phi\left(-\frac{c_m'(\theta)F_\theta(t) + b}{\theta(F_\theta(t))^{\frac{1}{2}}}\right)\right\}. \quad ///
\end{aligned}$$

Note that T may not be a proper random variable, i.e. $P_\theta(T = \infty)$ can be positive. When $c_m'(\theta) > 0$, $G_\theta(t)$ is very similar to the inverse Gaussian distribution function (Shuster (1968)).

The average sampling time (AST) for our sequential testing procedure is given by

$$\int_0^{t_0} t\,dG_\theta(t) + t_0(1 - G_\theta(t_0)). \tag{4.1}$$

The integral in (4.1) cannot be obtained in closed form, but is easily integrated computationally by the Gaussian or other quadratures.

5. THE INTERVAL ESTIMATION OF θ AFTER TESTING

For the completeness of this paper, we utilize Siegmund's (1978) method of obtaining a confidence interval of a parameter θ for an arbitrary parent distribution after a test of hypothesis of θ has been established. First, points on the stopping boundary are ordered in a counterclockwise direction. Then a lower $(1 - \gamma)$ confidence bound for θ is the smallest value of θ which gives probability at least γ to the event that the test terminates at a boundary point at least as large as that actually observed in the ordering. An upper confidence bound is defined similarly.

For our situation, an interval estimate of θ when H_0 is accepted is much more desirable in practice than when H_1 is accepted. When H_1 is accepted, further development and testing will be necessary so that it is more important to reach a rejection decision as soon as possible than to give an accurate estimate of θ. The following theorem gives a $(1 - 2\gamma)$ confidence interval for θ after H_0 is accepted. The proof of this theorem is straightforward.

<u>Theorem 4</u>. When H_0 is accepted, let the observed value of $\hat{\theta}_m(t_0)$ be d. Then a lower $(1 - \gamma)$ confidence bound is given by

$$\underline{\theta}(d) = \inf\{\theta: P_\theta(T \geq t_0 \text{ and } \hat{\theta}_m(t_0) \geq d) \geq \gamma\}; \tag{5.1}$$

and an upper $(1 - \gamma)$ confidence bound is given by

$$(5.2) \qquad \bar{\theta}(d) = \sup\{\theta: [P_\theta(T < t_0) + P_\theta(T \ge t_0 \text{ and } \hat{\theta}_m(t_0) < d)] \ge \gamma\}.$$

Finding the upper and lower bounds for θ involves computing the three probabilities in (5.1) and (5.2). Since $P_\theta(T \ge t_0 \text{ and } \hat{\theta}_m(t_0) \ge d)$ $= P_\theta(\hat{\theta}_m(t_0) \ge d) - P_\theta(T < t_0 \text{ and } \hat{\theta}_m(t_0) \ge d)$ and $P_\theta(T \ge t_0 \text{ and } \hat{\theta}_m(t_0) < d)$ $= P_\theta(\hat{\theta}_m(t_0) < d) - P_\theta(T < t_0) + P_\theta(T < t_0 \text{ and } \hat{\theta}_m(t_0) \ge d)$, the only quantity which we are unable to compute for large m from results in the previous sections is $P_\theta(T < t_0 \text{ and } \hat{\theta}_m(t_0) \ge d)$. The following corollary to previous results gives an approximate expression for this probability.

<u>Corollary 1</u>. When m is large,

$$P_\theta(T < t_0 \text{ and } \hat{\theta}_m(t_0) \ge d) = \int_0^{t_0} \Phi\left(\frac{c_m'(\theta)F_\theta(t) - d_m'(\theta)F_\theta(t_0) + b}{\theta(F_\theta(t_0) - F_\theta(t))^{\frac{1}{2}}}\right) g(t)dt,$$

where $d_m'(\theta) = m^{\frac{1}{2}}(d-\theta)$, $g(t) = \frac{dG(t)}{dt}$, G is defined in *Theorem* 3, and b is defined in Section 2.

<u>Proof</u>. Write $P_\theta(\hat{\theta}_m(t_0) \ge d \text{ and } T < t_0) = P_\theta(\hat{\theta}_m(t_0) \ge d \mid T < t_0)P_\theta(T < t_0)$

$$= \{E\, P_\theta(\hat{\theta}_m(t_0) \ge d \mid T, T < t_0)\}\ P_\theta(T < t_0)$$

$$= \int_0^{t_0} P_\theta(\hat{\theta}_m(t_0) \ge d \mid T = t,\ t < t_0) g(t)dt.$$

Now, letting $s = F_\theta(t)$, and $s_0 = F_\theta(t_0)$, we obtain

$$P_\theta(\hat{\theta}_m(t_0) \ge d \mid T = t, t < t_0) = P_\theta\left(\frac{\theta\rho(s_0)}{s_0} \ge d_m'(\theta) \mid \theta\rho(s) = sc_m'(\theta)\right)$$

$$= P_\theta(\rho(s_0) \ge s_0 d_m'(\theta)/\theta \mid \rho(s) = c_m'(\theta)s/\theta)$$

$$= P_\theta(\rho(s_0 - s) \ge (s_0 d_m'(\theta) - c_m'(\theta)s)/\theta \mid \rho(0) = b/\theta)$$

$$= P_\theta(\rho(s_0-s) \geq (s_0 d'_m(\theta)-c'_m(\theta)s-b)/\theta \mid \rho(0) = 0)$$

$$= \Phi\left(\frac{b-s_0 d'_m(\theta)+c'_m(\theta)s}{\theta\sqrt{s_0-s}}\right),$$

where we have used the stationary increments property of $\rho(s)$. ///

The integral in Corollary 1 can be easily computed by numerical integration procedures in practice, thus giving approximate values for (5.1) and (5.2) for large m.

6. AN EXAMPLE

In this section, we give a simple example of computing the initial value c_m,b, and the AST for illustration purposes. We let m = 100, $\theta_0 = 1.5$, $\theta_1 = 1.0$, $\alpha = .05$, and $\beta = .1$. *To attain given values* $\alpha = P_{\theta_0}$(reject H_0) and $\beta = P_{\theta_1}$(reject H_1), the pair of nonlinear equations in (3.1) are solved. If t_0 is preassigned $t_0 = 2.079$ such that $F_{\theta_0}(t_0) = .75$, then $c_m = 1.2469$ and b = 1.2436. We also report the test based on $\hat{\theta}_m(t^*)$ for a fixed length of time t* (Spurrier and Wei (1979)) for comparison purposes. For the same α and β values, t* = 1.0942. Table 1 gives the average sampling times (AST) of our sequential testing procedure for several θ values, computed by the 24-point Gaussian quadrature formulas. Under the alternative hypothesis, the AST's are considerably smaller than t* for the fixed time test. Under H_0, the AST is larger than t*. However, this is actually an advantage because we would like to have an accurate estimate of θ when H_0 is accepted.

Table 1. THE AVERAGE SAMPLING TIME

$t_0 = 2.079$, $t^* = 1.094$

θ	0.2	0.4	0.6	0.8	1.0	1.5
AST	0.017	0.060	0.129	0.269	0.779	2.009

7. REMARKS

In practice, we perform the sequential procedure proposed in this article by computing $\hat{\theta}_m(t)$ periodically with small increments of time. By doing this our test can be treated as an approximation to repeated significance tests with fixed length of time.

When t_0 is not predetermined by the experimenter, a value of t_0 can be obtained which is optimal in some sense. For example, t_0 may be chosen to minimize the AST when $\theta = \theta_0$ subject to constraints (3.1).

If our purpose were to obtain a decision as soon as possible without regard to estimation, then other sequential tests based on $\hat{\theta}_m$ could be constructed to meet this requirement.

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Let m items be put on test at the outset, and suppose an item is not replaced upon failure. Assume an exponential failure distribution $F_\theta(t) = 1 - \exp(-t/\theta)$. A time truncated sequential procedure for testing $H_0: \theta \geq \theta_0$ versus $H_1: \theta \leq \theta_1$ is developed. This procedure allows a quick rejection of H_0 when H_1 is true, but provides an accurate interval estimate of θ when H_0 is accepted after the test has been established.

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